CLASS: M.Sc. MATHEMATICS **15A/ 322**

St. JOSEPH'S COLLEGE (AUTONOMOUS) TIRUCHIRAPPALLI – 620 002

SEMESTER EXAMINATIONS – APRIL 2015

TIME: 3 Hrs. **MAXIMUM MARKS: 100**

SEM	SET	PAPER CODE	TITLE OF THE PAPER
IV	2013	12PMA4203A	ALGEBRAIC NUMBER THEORY

SECTION – A

Answer all the questions: **10 x 2 = 20**

1. If $a^n \equiv 1 (\text{mod } m)$ and d is the order of a modulo m, prove that d divides n.

2. Find $\phi(4)$ and $\phi(8)$.

3. Solve: $11x – 33y = 22$.

4. Solve: $3x + 2 \equiv 0 \pmod 7$

5. Find the primitive roots of 7.

6. If r is the order of a (mod m), prove that r divides $\phi(m)$.

7. If a is R and $a \equiv b \pmod p$, prove that b is R.

8. If p is an odd prime, show that $\sum_{a=1}^{p-1} \left[\frac{a}{p}\right]$

9. Find $\left[\frac{5}{17}\right]$.

10. Show that the congruence $x^2 \equiv 15 \pmod{1093}$ has no solution.

SECTION – B

Answer all the questions: **5 x 7 = 35**

11. a. Find the smallest value of $|36^m - 5^n|$ where m,n are natural numbers.

OR

b. State and prove Euler's theorem.

12. a. Solve: $738x + 621y = 45$

OR

b. Solve: $x + 2y + 3z = 1$

13. a. If the order of a and b modulo m are λ and μ respectively and $(\lambda, \mu) = 1$ prove that the order of ab modulo m is $\lambda\mu$.

OR

b. Using the method of indices, solve: $11x \equiv 5 \pmod{13}$

14. a. State and prove Euler's criterion.

OR

b. Show that $\left[\frac{2}{p}\right] = (-1)^{\frac{p^2-1}{8}}$

15. a. Determine $\left[\frac{-42}{31}\right]$

OR

b. Show that the congruence $x^2 + 23 \equiv 0 \pmod{59}$ has solution.

SECTION – C

Answer any THREE questions: **3 x 15 = 45**

16. a. State and prove Wilson's theorem. (8)

    b. Show that $(m-1)! \equiv (m-1) \bmod (1+2+...+(m-1))$ if and only if m is prime. (7)

17. a. State and prove Chinese remainder theorem. (8)

    b. Using Chinese remainder theorem, find the least natural number which when divided by 7, 10 and 11 leaves in order the remainders 1,6 and 2. (7)

18. a. If $m = p_1^{\alpha_1}.p_2^{\alpha_2}...p_k^{\alpha_k}$ has a primitive root prove that m must be of the form 2^k, p^k, $2p^k$. (8+7)

    b. Using the theory of indices, solve: $5x^2 + 3x - 10 \equiv 0 \pmod{13}$

19. a. State and prove Gauss Lemma. (8)

    b. Determine $\left[\frac{3}{p}\right]$ (7)

20. a. State and prove quadratic reciprocity law. (8)

    b. Find all primes for which 5 is a quadratic residue. (7)

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