CLASS: M.Sc. MATHEMATICS **15A/ 319**

St. JOSEPH'S COLLEGE (AUTONOMOUS) TIRUCHIRAPPALLI – 620 002

SEMESTER EXAMINATIONS – APRIL 2015

TIME: 3 Hrs. **MAXIMUM MARKS: 100**

SEM	SET	PAPER CODE	TITLE OF THE PAPER
IV	2013	12PMA4113	FUNCTIONAL ANALYSIS

SECTION – A

Answer all the questions: **10 x 2 = 20**

1. Define normed linear space.
2. Define dual space with example.
3. Show that any finite dimensional space is reflexive.
4. Define Minkowski functional of K.
5. Define uniformly bounded.
6. Define open mapping.
7. Give the proof of Pythagorean theorem.
8. Define an orthonormal set.
9. Define unitary operator.
10. Define orthogonal projection.

SECTION – B

Answer all the questions: **5 x 7 = 35**

11. a. Define Schauder basis. Prove that a Banach space having a Schauder basis is separable.

OR

b. Let X = K^n, Y = K^m. So that any $T \in BL(X,Y)$ is represented by m×n matrix (α_{ij}). If X,Y are endowed with the norm $\| \cdot \|_m$ show that $\|T\| = \max\limits_{1 \le i \le m} \left(\sum\limits_{j=1}^{n} |\alpha_{ij}| \right)$.

12. a. Let X be a reflexive Banach space. Then prove that every closed subspace of X is reflexive.

OR

b. Prove that the natural embedding J of a normed linear space X into its second dual X^{**} is linear and preserves the norm.

13. a. In l_p space for $1 < p < \infty$, prove that $x_n \xrightarrow{w} x_0$ iff $\{x_n\}$ is bounded and $x_n \to x_0$ co-ordinate wise.

OR

b. If X and Y are Banach spaces, then prove that X × Y is also a Banach space.

14. a. Prove that an inner product space X is a normed linear space with the norm defined by $\|x\| = (x, x)^{1/2}$.

OR

b. State and prove Bessel's inequality.

15. a. Let X be a Hilbert space and $T \in BL(X)$. Then the following are equivalent.

(i) T is isometric

(ii) $(Tx, Ty) = (x,y) \ \forall \ x, y \in X$

(iii) $T^*T = I$

OR

b. Let X be a Hilbert space and $T \in BL(X)$ self adjoint. Then prove that $\|T\| = \sup\limits_{\|x\|=1} |(Tx, x)|$.

SECTION – C

Answer any THREE questions: **3 x 15 = 45**

16. Let X,Y be normed linear spaces over K (K means R or C) and $T: X \rightarrow Y$ a linear mapping. Prove that the following are equivalent.
    (i) T is continuous at a point
    (ii) T is continuous on X.
    (iii) T is uniformly continuous
    (iv) T is bounded
    Further if Y = K, then all these are equivalent to
    (v) the nullspace $N(T) = \{x \in X / Tx = 0\}$ is closed in X.

17. State and prove separation form of Hahn – Banach theorem.

18. State and prove open mapping theorem.

19. Let M be a closed subspace of a Hilbert space X. Then prove that $X = M \oplus M^{\perp}$. Also prove that $M = M^{\perp\perp}$.

20. Let $P_1, \ldots P_n$ be the orthogonal projections on $M_1, \ldots M_n$ respectively and let $M_j \perp M_k$ for $j \neq k$. Then prove that $P = P_1 + P_2 + \ldots P_n$ is the orthogonal projection on $M = M_1 + \ldots + M_n$.

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