

SEM	SET	PAPER CODE	TITLE OF THE PAPER
II	2014	14PMA2108	COMPLEX ANALYSIS

SECTION – B**Answer all the questions:****5 x 5 = 25**

31. a. Assume that $f(z)$ is analytic and satisfies the inequality $|f(z)-1| < 1$ in a region Ω . Show that $\int \frac{f'(z)}{f(z)} dz = 0$ for every closed curve in Ω .

OR

- b. State and prove Cauchy's Theorem in a disk.
32. a. State and prove the Cauchy's integral formula.

OR

- b. If $P(z)$ is a polynomial of degree n , $n \geq 1$ with real or complex co-efficient then the equation $P(z) = 0$ has at least one root.
33. a. State and prove Cauchy's residue theorem.

OR

- b. Let γ be homologous to zero in Ω and such that $n(\gamma, z)$ is either 0 or 1 for any z not on γ . Suppose $f(z)$ and $g(z)$ are analytic in Ω and satisfy the inequality $|f(z) - g(z)| < |f(z)|$ on γ . Prove that $f(z)$ and $g(z)$ have the same number of zeros enclosed by γ .
34. a. State and prove the mean value property.

OR

- b. State and prove the Schwarz theorem.

35. a. Prove that the zero a_1, a_2, \dots, a_n and poles b_1, b_2, \dots, b_n of an elliptic function satisfy $a_1 + a_2 + \dots + a_n \equiv b_1 + b_2 + \dots + b_n \pmod{M}$.

OR

- b. Prove that the sum of an elliptic function is zero.

SECTION – C

Answer any THREE questions:

3 x 15 = 45

36. State and prove Cauchy's Theorem for a rectangle.
37. Derive the higher derivatives formula.
38. Evaluate $\int_0^{\infty} \frac{dx}{x^4 + a^4}$, ($a > z$)
39. State and prove Laurent's Series.
40. Prove that a discrete module consists either of zero alone, of the integral multiples of a single complex number $w \neq 0$ or of all linear combinations $n_1 w_1 + n_2 w_2$ with integral co-efficients of two numbers w_1, w_2 with non-real ratio w_2/w_1 .
