CLASS: M.Sc. MATHEMATICS **15A/ 317**

St. JOSEPH'S COLLEGE (AUTONOMOUS) TIRUCHIRAPPALLI – 620 002

SEMESTER EXAMINATIONS – APRIL 2015

TIME: 2 Hrs. 20 Mins. **MAXIMUM MARKS: 70**

SEM	SET	PAPER CODE	TITLE OF THE PAPER
II	2014	14PMA2107	ALGEBRA

SECTION – B

Answer all the questions: **5 x 5 = 25**

31. a. Prove that the subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

OR

b. State and prove second part of Sylow's Theorem.

32. a. Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then prove that R is a field.

OR

b. Let R be a Elclidean ring. Prove that any two elements a and b in R have a greatest common divisor d moreover $d = \lambda a + \mu b$ for some $\lambda, \mu \in R$.

33. a. Given two polynomials f(x) and $g(x) \neq 0$ in F[x], prove that there exist two polynomials t(x) and r(x) in F[x] such that $f(x) = t(x)\, g(x) + r(x)$ where $r(x) = 0$ or $\deg r(x) < \deg g(x)$.

OR

b. If f(x) and g(x) are primitive polynomials, prove that f(x) g(x) is a primitive polynomial.

34. a. Let $f(x) \in F[x]$ be of degree $n \geq 1$, prove that there is an extension E of F of degree at most n! in which f(x) has n roots.

OR

b. If the polynomial $f(x) \in F[x]$ has a multiple root, prove that $f(x)$ and $f'(x)$ have a non trivial (ie., of positive degree) common factor.

35. a. If K is a finite extension of F, prove that G(K, F) is a finite group and its order, o(G(K,F)) satisfied $o(G(K,F)) \leq [K:F]$.

OR

b. Let F be a finite field with q elements and suppose that $F \subset K$ where K is also a finite field. Prove that K has q^n elements where $n = [K:F]$.

SECTION – C

Answer any THREE questions: 3 x 15 = 45

36. a. Let ϕ be a homomorphism of G onto $\overline{G}$ with kernel K. Prove that $G/K \simeq \overline{G}$.

b. State and prove Cauchy's theorem.

37. Prove that J[i] is a Euclidean ring.

38. i. State and prove the Einstein Criterion. (8)

ii. If R is a unique factorization domain and if p(x) is a primitive polynomial in R[x], prove that it can be factored in a unique way as the product of irreducible elements in R[x]. (7)

39. If L is a finite extension of K and if K is a finite extension of F, prove that L is a finite extension of F. Moreover,

[L:F] = [L:K] [K:F].

40. i. Prove that the fixed field of G is a subfield of K. (5)

ii. K is a normal extension of F iff K is splitting field of some polynomial over F. (10)

****