CLASS: M.Sc. MATHEMATICS **15A / 317**

St. JOSEPH'S COLLEGE (AUTONOMOUS) TIRUCHIRAPPALLI – 620 002

SEMESTER EXAMINATIONS – APRIL 2015

TIME: 40 Minutes **MAXIMUM MARKS: 30**

SEM	SET	PAPER CODE	TITLE OF THE PAPER
II	2014	14PMA2107	ALGEBRA

SECTION - A

Answer all the questions: **$30 \times 1 = 30$**

Choose the correct answer:

1. N is a normal subgroup of G if and only if
   a) $gNg^{-1} = \phi \forall_g \in G$ b) $gNg^{-1} = N$, for every $g \in G$
   c) $gNg^{-1} \subset G \ \forall g \in G$ d) $gNg^{-1} = n \ \forall g \in G$
2. If G is a group, N a normal subgroup of G. Then G/N is also
   a) a set b) a normal subgroup
   c) a group d) a subgroup
3. A mapping ϕ from a group G into a group $\overline{G}$ is said to be a homomorphism.
   a) if for all $a,b \in G$, $\phi(ab) = \phi(a)\,\phi(b)$
   b) if for all $a,b \in G$, $\phi(ab) \neq \phi(a)\,\phi(b)$
   c) if for each $a,b \in G$, $\phi(ab) = \phi(ba)$
   d) if for each $a \in G$, $\phi(a) = \bar{e}$
4. If $a,b \in G$, then b is said to be a conjugate of a in G,
   a) if there exists an element $e \in G$, such that $b = aea^{-1}$
   b) if there exists an element $c \in G$, such that $a = c^{-1}ac$
   c) if there exists an element $c \in G$, such that $c^{-1} = abc$
   d) if there exists an element $c \in G$, such that $b = c^{-1}ac$

5. Let p(n) denote the no. of partition of n. Then p(5) =

a) 3 b) 4

c) 7 d) 11

6. If A and B are finite subgroup of G. Then

a) $o(AxB) = \frac{o(A) \cap o(B)}{o(A \cap xBx^{-1})}$ b) $o(A_x B) = \frac{o(A)o(B)}{o(A \cap xBx^{-1})}$

c) $o(A_x B) = \frac{o(A) \cup o(B)}{o(A \cup xBx^{-1})}$ d) $o(A_x B) = \frac{o(A)o(B)}{o(A \cup xBx^{-1})}$

7. If U is an ideal of R and $1 \in U$. Then

a) U = R/U b) $U \neq R$

c) U = R d) none of the above

8. An ideal $M \neq R$ is a ring R is said to be a maximal ideal of R if

a) whenever U is an ideal of R such that $M \subset U \in R$, then either R = U or M = U

b) whenever U is an ideal of R such that $R \subset U \in M$, then either R = U and M = R

c) whenever U is an ideal of R such that R = U = M

d) none of the above

9. A ring R can be imbedded in a ring R^1 if

a) $ab = 0 \ \forall \ a,b \in R$ b) there is an isomorphism of R into R^1

c) it has zero divisions d) none of these

10. If K is any field which contains D, then

a) D is a commutative ring with unit element

b) there is an isomorphism of D into K

c) K does not contain a subfield to K

d) K contains a subfield isomorphic to K

11. An integral domain R with unit element is a principal idle ring

a) if all ideals A in R is of the form A = (a) for some $a \in R$

b) if every ideal A in R is of the form A = (a) for all $a \in R$

c) if every ideal A in R is of the form A = (a) for some $a \in R$

d) if every ideal A in R is of the form A = (a) for some $a \in A$

12. If $x = a+bi \in J[i]$, then d(x) =

a) $\sqrt{a^2+b^2}$

b) $a^2 + b^2$

c) $a^2 - b^2$

d) $\sqrt{a^2-b^2}$

13. If f(x), g(x) are two non zero elements of F[x], then deg(f(x)g(x)) is equal to

a) deg f(x) – deg g(x)

b) deg f(x) + i deg g(x)

c) deg f(x) + deg g(x)

d) –deg f(x) – deg g(x)

14. Any polynomial in F[x] can be written in a unique manner as a

a) product of irreducible polynomials in F[x]

b) sum of irreducible polynomials in F[x]

c) difference of irreducible polynomials in F[x]

d) division of irreducible polynomials in F[x]

15. The polynomial $f(x) = a_0 + a_1x + ... + a_nx^n$, where $a_0, a_1,...a_n$ are integers is said to be primitive if the g.c.d of $a_0, a_1,...a_n$ is

a) 0

b) 2

c) –1

d) 1

16. A polynomial is said to be integer monic if all its coefficients are integers and its

a) value is 1

b) lowest coefficient is 1

c) highest coefficient is 1

d) highest coefficient is zero

17. R[x] is

a) a commutative ring with zero element

b) a commutative ring with unit element

c) not a commutative ring with unit element

d) not an integral domain

18. If $a \in R$ is an irreducible element and a|bc, then

a) a|b or a|c   b) a|b and a|c

c) b|a or c|a   d) c|a and b|a

19. Let F be a field, a field K is said to be an extension of F

a) if F contains K   b) if K contains F

c) if F does not contain K   d) if K does not contain F

20. A complex number is said to be an algebraic number if it is

a) algebraic over the field of complex numbers

b) algebraic over the field of positive numbers

c) algebraic over the field of negative numbers

d) algebraic over the field of rational numbers

21. An extension K of F is a simple extension if

a) $K = F(\alpha)$ for some $\alpha \in K$ b) [K:F] is finite

c) $F = K(\alpha)$ for some $\alpha \in F$ d) $K_{G(K,F)} = F$

22. F(a) is

a) finite extension of F

b) a smallest field that contains a and F

c) F(a) is an algebraic extension of F

d) F(a) is field that contains a and F

23. If $p(x) \in F[x]$, then an element a in some extension field of F is called

a) a root of p(x) if p(a)=0   b) a root of p(x) if $p(a) \neq 0$

c) not a root of p(x) if p(a)=0   d) not a root of p(x) if $p(a) \neq 0$

24. The element $a \in K$ is a root of $p(x) \in F[x]$ of multiplicity m if

a) $(x+a)^m \nmid p(x)$, where as $(x-a)^{m+1} \nmid p(x)$

b) $(x-a)^m \nmid p(x)$ where as $(x-a)^{m+1} \nmid p(x)$

c) $(x-a)^m \nmid p(x)$ where as $(x-a)^{m+1} \nmid p(x)$

d) $(x-a)^{m+1} \mid p(x)$ where as $(x-m)^m \nmid p(x)$

25. The automorphism σ of K is in G (K, F) if and only if

a) $\sigma(\alpha)=\alpha$ for every $\alpha \in F$

b) $\sigma(\alpha\beta) = \sigma(\alpha)\,\sigma(\beta), \forall \alpha, \beta \in F$

c) $\sigma(\alpha \pm \beta) = \sigma(\alpha) \pm \sigma(\beta), \forall \alpha, \beta \in K$

d) $\sigma(\alpha) = 0 \, \forall \alpha \in F$

26. If σ is an automorphism on K and $\sigma^* = \sigma / T$, where T is a subfield of K, then σ^* is

a) a homomorphism

b) an automorphism

c) 1–1 homomorphism

d) onto homomorphism

27. If K is a normal extension of F, then

a) F is the splitting field of some polynomial over K

b) K is the splitting field of some polynomial over F

c) F is the splitting field of some rational over K

d) K is the splitting field of some rational over F

28. If the finite field F has p^m elements, then

a) every $a \in F$ satisfies $a^{p^m - 1} = 0$

b) every $a \in F$ satisfies $a^{p^m} = 1$

c) every $a \in F$ satisfies $a^{p^m} = a$

d) every $a \in F$ satisfies $a^{p^m} = a^2$

29. For every prime number p and every positive integer m there is a unique field having

a) p^{m-1} elements b) p^m elements

c) p^{2m} elements d) $p^m - 1$ elements

30. Any generator of a cyclic group under multiplication is called a

a) primitive root of p b) characteristics roots of p

c) polynomial in p d) finite field

****