CLASS: M.Sc. MATHEMATICS **15A/ 316**

St. JOSEPH'S COLLEGE (AUTONOMOUS) TIRUCHIRAPPALLI – 620 002

SEMESTER EXAMINATIONS – APRIL 2015

TIME: 2 Hrs. 20 Mins. **MAXIMUM MARKS: 70**

SEM	SET	PAPER CODE	TITLE OF THE PAPER
II	2014	14PMA2106	REAL ANALYSIS – II

SECTION – B

Answer all the questions: **5 x 5 = 25**

31. a. State and prove the chain rule for differentiation.

OR

b. Let f be defined on [a,b]; if f has a local maximum at a point $x \in (a,b)$, and if $f'(x)$ exists, prove that $f'(x) = 0$.

32. a. Prove that $f \in \Re(\alpha)$ on [a,b] if and only if for every $\varepsilon > 0$ there exists a partition P such that $U(P, f, \alpha), L(P, f, \alpha) < \varepsilon$.

OR

b. Let $f \in \Re(\alpha)$ on [a,b]. For $a \le x \le b$. put $F(x) = \int_a^x f(t)dt$. Prove that F is continuous on [a,b]; further more, if f is continuous at a point x_0 of [a,b], then F is differentiable and $F'(x_0) = f(x_0)$.

33. a. State and prove Cauchy criterion for uniform convergence of sequence of functions.

OR

b. Let α be monotonically increasing on [a,b]. Suppose $f_n \in \Re(\alpha)$ on [a,b], for n=1,2,3,.., and suppose $f_n \to f$ uniformly on [a,b], prove that $f \in \Re(\alpha)$ on [a,b] and $\int_a^b f \, d\alpha = \lim_{n\to\infty} \int_a^b f_n \, d\alpha$.

34. a. State and prove Taylor's Theorem on the expansion of power series.

OR

b. If, for some x, there are constants $\delta > 0$ and $M < \infty$ such that $|f(x+t) - f(x)| \le M|t|$ for all $t \in (-\delta, \delta)$, prove that

$$\lim_{N\to\infty} S_N(f;x) = f(x).$$

35. a. Let r be a positive integer. If a vector space X is spanned by a set of r vectors, prove that dim $X \le r$.

OR

b. Suppose E is an open set in R^n, f maps E into R^m, f is differentiable at $x_0 \in E$, g maps an open set containing f(E) into R^k, and g is differentiable at $f(x_0)$. Prove that the mapping F of E into R^k defined by $F(x) = g(f(x))$ is differentiable at x_0, and $F'(x_0) = g'(f(x_0))f'(x_0)$.

SECTION – C

Answer any THREE questions: **3 x 15 = 45**

36. State and prove L'Hospital's rule.

37. Assume α increases monotonically and $\alpha' \in \Re$ on [a,b]. Let f be a bounded real function on [a,b]. Prove that $f \in \Re(\alpha)$ and $\int_a^b f \, d\alpha = \int_a^b f(x)\, \alpha'(x) dx$.

38. Suppose $\{f_n\}$ is a sequence of functions, differentiable on [a,b] and such that $\{f_n(x_0)\}$ converges for some point x_0 on [a,b]. If $\{f_n'\}$ converges uniformly on [a,b], prove that $\{f_n\}$ converges uniformly on [a,b] to a function f and $f'(x) = \lim_{n\to\infty} f_n'(x) \quad (a \le x \le b)$.

39. State and prove Parseval's theorem.

40. State and prove Inverse function theorem.

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