CLASS: M.Sc. MATHEMATICS **15A / 316**

St. JOSEPH'S COLLEGE (AUTONOMOUS) TIRUCHIRAPPALLI – 620 002

SEMESTER EXAMINATIONS – APRIL 2015

TIME: 40 Minutes **MAXIMUM MARKS: 30**

SEM	SET	PAPER CODE	TITLE OF THE PAPER
II	2014	14PMA2106	REAL ANALYSIS – II

SECTION - A

Answer all the questions: **30 × 1 = 30**

Choose the correct answer:

1. If f is derivable in (a,b) and $f'(x) \geq 0$ for all $x \in (a,b)$, then f is

   a) constant b) monotonically decreasing

   c) monotonically increasing d) none of the above

2. $\lim\limits_{t \to x} \frac{f(t) - f(x)}{t - x}$ is equal to

   a) $f'(t)$ b) $f'(x)$

   c) 0 d) $(t-x) f'(x)$

3. The function f defined on [a,b] has a local maximum at a point $x \in (a,b)$ and if $f'(x)$ exists, then

   a) $f'(x) > 0$ b) $f'(x) < 0$

   c) $f'(x) = 0$ d) $f'(x) = x$

4. Suppose f is a real differentiable function on [a,b] and suppose $f'(a) < \lambda < f'(b)$, then there is a point $x \in (a,b)$ such that

   a) $f'(x) = f(a)$ b) $f'(x) = f(b)$

   c) $f'(x) = f(\lambda)$ d) $f'(x) = \lambda$

5. $U(P, f, \alpha)$ is equal to

a) $\sum_{i=1}^{n} M_i(\alpha(x_i - x_{i-1}))$

b) $\sum_{i=1}^{n} M_i(\alpha(x_i) - \alpha(x_{i-1}))$

c) $\sum_{i=1}^{n} m_i(\alpha(x_{i-1} - x_i))$

d) $\sum_{i=1}^{n} M_i(\alpha(x_{i-1} - x_i))$

6. Which one of the following is not true?

a) $\overline{\int_a^b} f d\alpha \leq U(P, f, \alpha)$

b) $\underline{\int_a^b} f d\alpha \geq U(P, f, \alpha)$

c) $\underline{\int_a^b} f d\alpha \geq \overline{\int_a^b} f d\alpha$

d) $\underline{\int_a^b} f d\alpha \leq \overline{\int_a^b} f d\alpha$

7. $f \in \Re(\alpha)$ if

a) $\overline{\int_a^b} f d\alpha$ exists

b) $\underline{\int_a^b} f d\alpha$ exists

c) $\int_a^b f d\alpha$ exists

d) $\overline{\int_a^b} f d\alpha$ and $\underline{\int_a^b} f d\alpha$ exists

8. Two refinements of $P_1 \cap P_2$ are

a) P_1 and P_2

b) P_1 and P_2^C

c) P_1^C and P_2^C

d) $P_1 \cup P_2$

9. If P^* is a common refinement of P_1 and P_2, then

a) $L(P_1, f, \alpha) \geq L(P^*, f, \alpha) \geq U(P^*, f, \alpha) \geq U(P_2, f, \alpha)$

b) $L(P_1, f, \alpha) \leq L(P^*, f, \alpha) \leq U(P^*, f, \alpha) \leq U(P_2, f, \alpha)$

c) $L(P_1, f, \alpha) \leq U(P^*, f, \alpha) \leq U(P_2, f, \alpha) \leq L(P^*, f, \alpha)$

d) $L(P_1, f, \alpha) \leq U(P^*, f, \alpha) \leq L(P^*, f, \alpha) \leq U(P_2, f, \alpha)$

10. If f is monotonically increasing on [a,b], then

a) $m_i = f(x_1)$

b) $m_i = f(x_{i-1})$

c) $m_{i-1} = f(x_{i-1})$

d) $m_{i-1} = f(x_1)$

11. If $f_n(x) = 1 + x^n$, $x \in (0,1)$ then $\lim_{n\to\infty} f_n(x)$ is equal to

a) 0

b) 1

c) ∞

d) none

12. Suppose $\lim_{n\to\infty} f_n(x) = f(x)$ and $M_n = \sup_{x\in E} |f_n(x) - f(x)|$. Then $f_n \to f$ uniformly if and only if

a) $M_n \to f$

b) $M_n \to 0$

c) $M_n \to 1$

d) $M_n \to x$

13. A sequence of functions $\{f_n\}$, n = 1,2,... converges uniformly on E to a function f if for every $\varepsilon > 0$ there is an integer N such that $n \geq N$ implies

a) $|f_n(x) - f(x)| \leq \varepsilon$ for all $x \in E$

b) $|f_n(x) - f(x)| \leq \varepsilon$ for all $x \in E$

c) $|f_n(x)| < |f(x)|$ for all $x \in E$

d) $|f_n(x)| < x$ for all $x \in E$

14. The complete metric space C(X) contains complex valued functions which are

a) continuous

b) bounded

c) differentiable

d) continuous and bounded

15. $\lim_{n\to\infty} \lim_{m\to\infty} \frac{m}{m+n} = \ldots$

a) 0

b) ∞

c) 1

d) m

16. A function f is continuous at x means $\lim_{t \to x} f(t) = \ldots$

a) f(x)
b) $f'(x)$
c) f(t)
d) $f'(t)$

17. The series $\Sigma f_n(x)$ converges uniformly on E if the sequence $\{s_n\}$ where s_n = ... converges uniformly on E.

a) $\sum_{i=1}^{\infty} f_n(x)$
b) $\sum_{i=1}^{n} f_n(x)$
c) $\sum_{i=n}^{\infty} f_n(x)$
d) $f_n(x)$

18. Suppose $\{f_n\}$ is a sequence of functions differentiable on [a,b]. If $\{f_n'\}$ converges uniformly on [a,b], then $\{f_n\}$ converges uniformly to f on [a,b] and $f'(x)$ = ...

a) $\lim_{n \to \infty} f_n(x)$
b) $\lim_{n \to \infty} f_n'(x)$
c) $\Sigma f_n(x)$
d) $\Sigma f_n'(x)$

19. Suppose $f(x) = \sum_0^{\infty} c_n x^n$ converges for |x| < R and f has derivatives of all orders given by

$f^{(k)}(x) = \sum_{n=k}^{\infty} n(n-1)\ldots(n-k+1)C_n x^{n-k}$, then $f^{(k)}(0)$ = ...

a) k!
b) $C_k k!$
c) 0
d) n!

20. Suppose $\sum c_n$ converges and $f(x) = \sum_{n=0}^{\infty} c_n x^n, -1 < x < 1$. Then $\lim_{x \to 1} f(x) = \ldots$

a) $\sum_{n=0}^{\infty} c_n$
b) $\sum_{n=0}^{\infty} x^n$
c) 1
d) 0

21. $E(Z) = \sum_{n=0}^{\infty} \frac{z^n}{n!}$ then E(n) = ...

a) e
b) n
c) e^n
d) n!

22. $\frac{1}{2\pi} \int_{-\pi}^{\pi} e^{inx} dx = ...$, for n = ±1, ±2...

a) 1
b) 0
c) n
d) e^n

23. For n = 1,2,3,..., $\int_0^{\infty} t^n e^{-t} dt = ...$

a) n!
b) (n – 1)!
c) e^n
d) n

24. Γ (½) =

a) π
b) $\sqrt{\pi}$
c) $\sqrt{2}$
d) $\sqrt{\frac{1}{2}}$

25. If $D_N(x) = \sum_{-N}^{N} e^{inx}$, then $\frac{1}{2\pi} \int_{-\pi}^{\pi} D_N(x) dx =$

a) 1
b) 0
c) π
d) $\sqrt{2}$

26. If the relation $c_1x_1 + c_2x_2 + ... + c_nx_n = 0$ of vector $x_1, x_2, ... x_n$ implies $c_1 = ... = c_n = 0$, then $\{x_1, x_2, ... x_n\}$ is

a) linearly independent
b) linearly dependent
c) empty set
d) vector space

27. Suppose X is a vector space of dimension n, then every basis of X consists of ______.

a) n vectors
b) < n vectors
c) > n vectors
d) only one vector

28. Let X be a metric space with metric d. The mapping ϕ from X into X is a contraction of X into X if there is a number ______ such that $d(\phi(x), \phi(y)) \leq cd(x,y)$ for all $x,y \varepsilon X$.

a) $c = 1$
b) $c = 0$
c) $c > 1$
d) $c < 1$

29. If X is a metric space and if ϕ is a contraction of X into X, then there is _____ $x \in X$ such that $\phi(x) = x$.

a) one and only one
b) more than one
c) less than one
d) no

30. If X is an n-dimensional vector space, then a set E of n vectors in X spans X if and only if ______.

a) E is independent
b) E is dependent
c) E is a vector space
d) None of the above

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